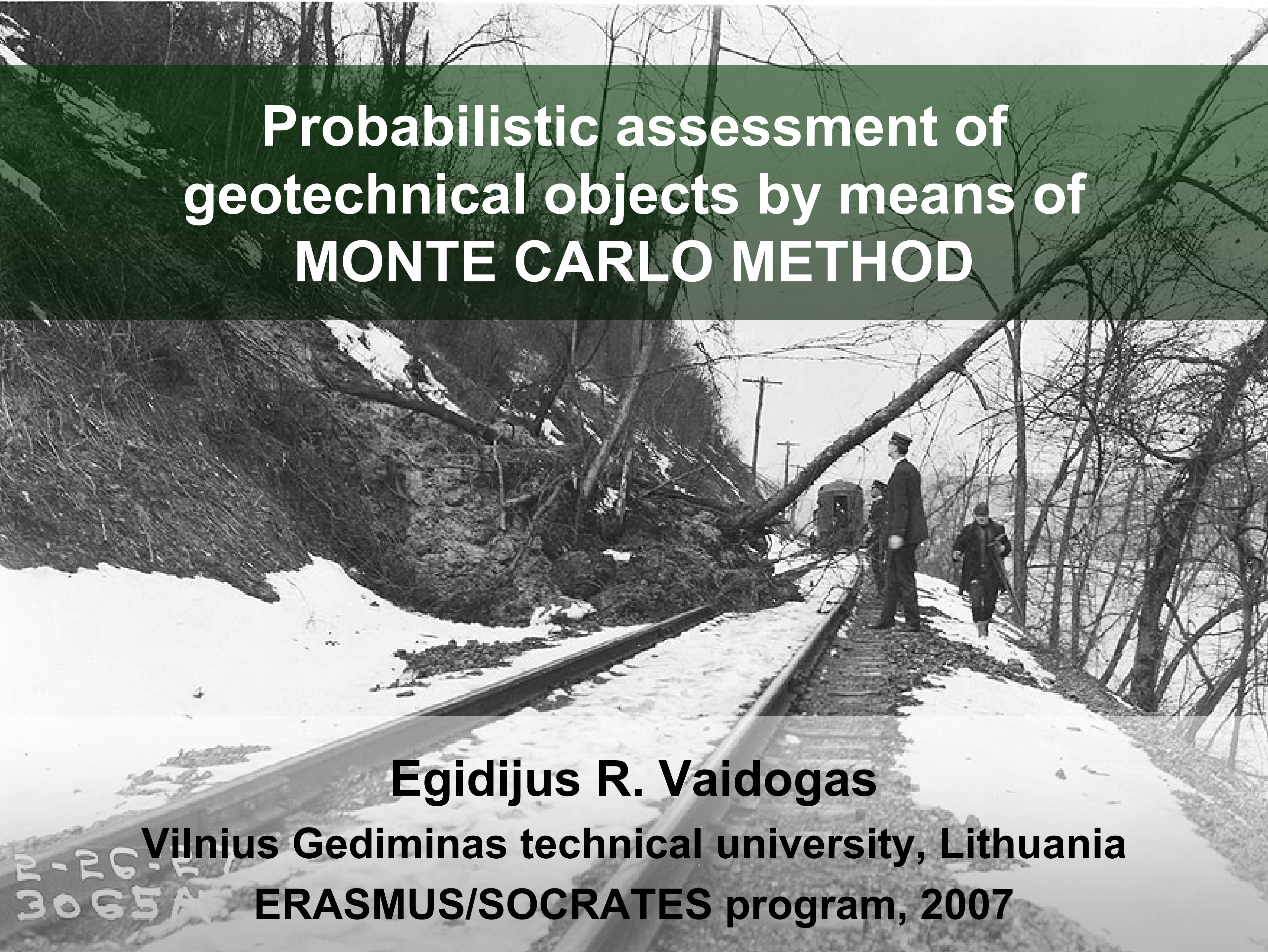
Probabilistic assessment of geotechnical objects by means of MONTE CARLO METHOD



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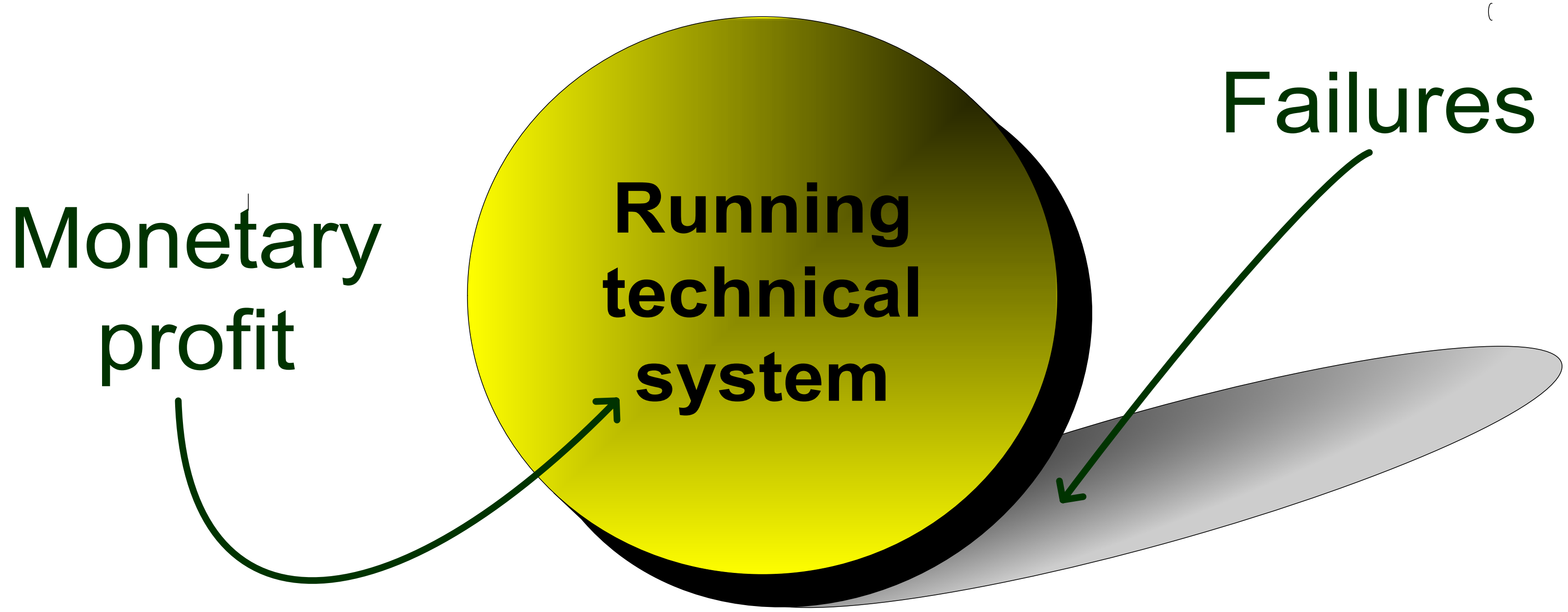
ERASMUS/SOCRATES program, 2007

What is the principal objective?

If you think that safety is expensive, you should try to have an accident !



To be brief…

GEOTECHNICAL OBJECT = STRUCTURE

Definition derived from EN 1990:2002

Structure = organised combination of connected parts, including fill placed during execution of the construction works, designed to carry loads and provide adequate rigidity

- Foundations
- Anchorages
- Retaining structures
- Embankments
- Slopes
- Excavations

CONTENTS OF LECTURE

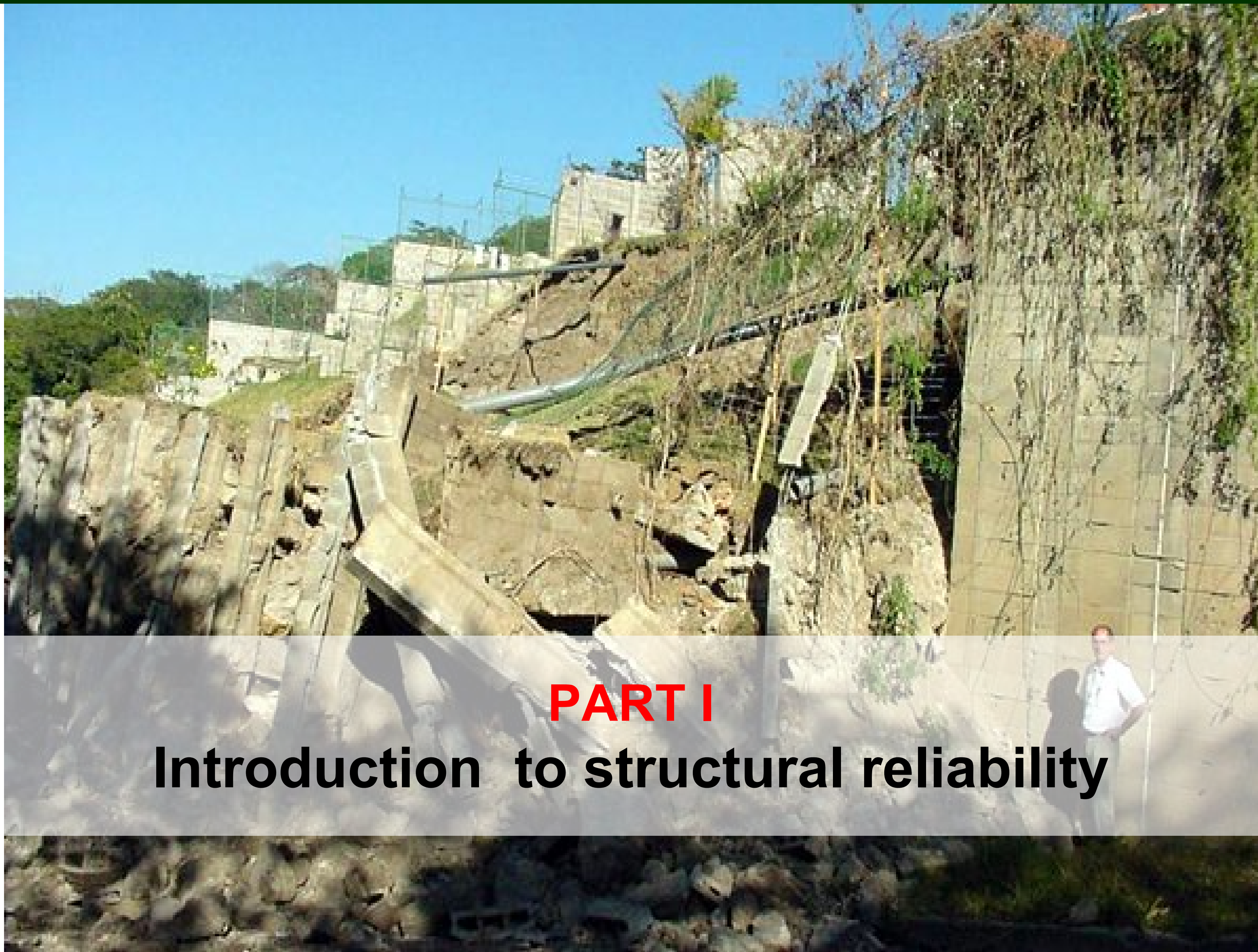
I. THEORETICAL PART

1. Introduction to structural reliability
2. Reliability & *failure probability* of structures
3. Estimation of *failure probability* by means of Monte Carlo method

II. COMPUTER EXERCISES

1. Introduction to *AntHill* computer code
2. Stability of slope excavated in a clay layer
3. Variability of load capacity of a ground anchor
4. Reliability of concrete friction pile
5. Reliability of gravity retaining wall
6. Reliability of cantilever retaining wall

Introduction to structural reliability



PART I
Introduction to structural reliability

Failures of structures

Structures unfortunately fail …

Failure of a structure **= insufficient load bearing capacity or inadequate serviceability of a structure or structural element**

Failure **= exceedance of limit state**

Design situations*

Failure can take place in one of the three situations:

- ► Persistent design situations (situation of normal use)
- ► Transient design situations** (construction, repair, demolition)
- ► Accidental design situations (fire, explosion, impact)

* Are mentioned in Eurocode 7 (prEN 1997-1: 2001(E). Geotechnical design. Part 1: General rules)

** Only the transient design situations are mentioned in Eurocode 7

I. Structural reliability 3/10

The risk of death as a result of structural failure*

Activity	Approximate death rate $\times 10^{-9}$ deaths/hr exposure	Estimated typical exposure (hr/year)	Typical risk of death $\times 10^{-6}$/ year
Construction work	**70...200**	**2200**	**150...400**
Manufacturing	20	2000	40
Coal mining (UK)	210	1500	300
Building fires	**1...3**	**8000**	**8...24**
Air travel	1200	20	24
Car travel	700	300	200
Train travel	80	200	15
Structural failures	**0,02**	**6000**	**0,1**

*** Melchers, R. E. (1987) Structural reliability Analysis and Prediction. Chichester: Ellis Horwood/Wiley**

I. Structural reliability 4/10

Why should we be concerned about structural reliability

- ***Individuals:*** **involuntary of risk due to structural failures**

The risk levels for buildings and bridges are usually associated with *involuntary risk* and are much lower than the risk associated with voluntary activities (travel, mountain climbing, deep see fishing)

- ***Society*****: failure results in decrease of confidence in stability and continuity of one's surroundings**

Society is interested in structural reliability only in the sense that a structural failure with significant consequences shatters confidence in the stability and continuity of one’s surroundings

- ***Engineers*****: the need to apply novel structures and novel construction methods generates interest in safety**

Design, construction, and use of *new* or *particularly hazardous* systems should be of particular interest in their safety (new and unique bridge, new off-shore structure, NPP, chemical plant, liquefied gas depot)

I. Structural reliability 5/10

Human errors cause up to 95% of failures

Structural accidents: phase in which error occurred*

Phase	Percentage of cases (493 cases)	Percentage of total cost damage (493 cases)
Design	37	43
Construction	35	20
Design & construction	18	22
Occupation	5	11
Others	5	4

*** Hauser, H. (1979) Lessons from European failures, Concrete international, ACI, 11(12), p. 21-25.**

Failures of FORMULA 1 cars

TOP SPEED

MASS

AERODYNAMIC PROPERTIES

CHANCE OF

FAIL-SAFE BEHAVIOUR

(RELIABILITY)

Structural reliability theory: *arguments in favor*

analysis of
structures, say, EC7

Time-independent

analysis of
error-free structures

Assessment of

Incorporation of possibility of

Consideration of

rather than individual components

Consideration of

(accidental actions)

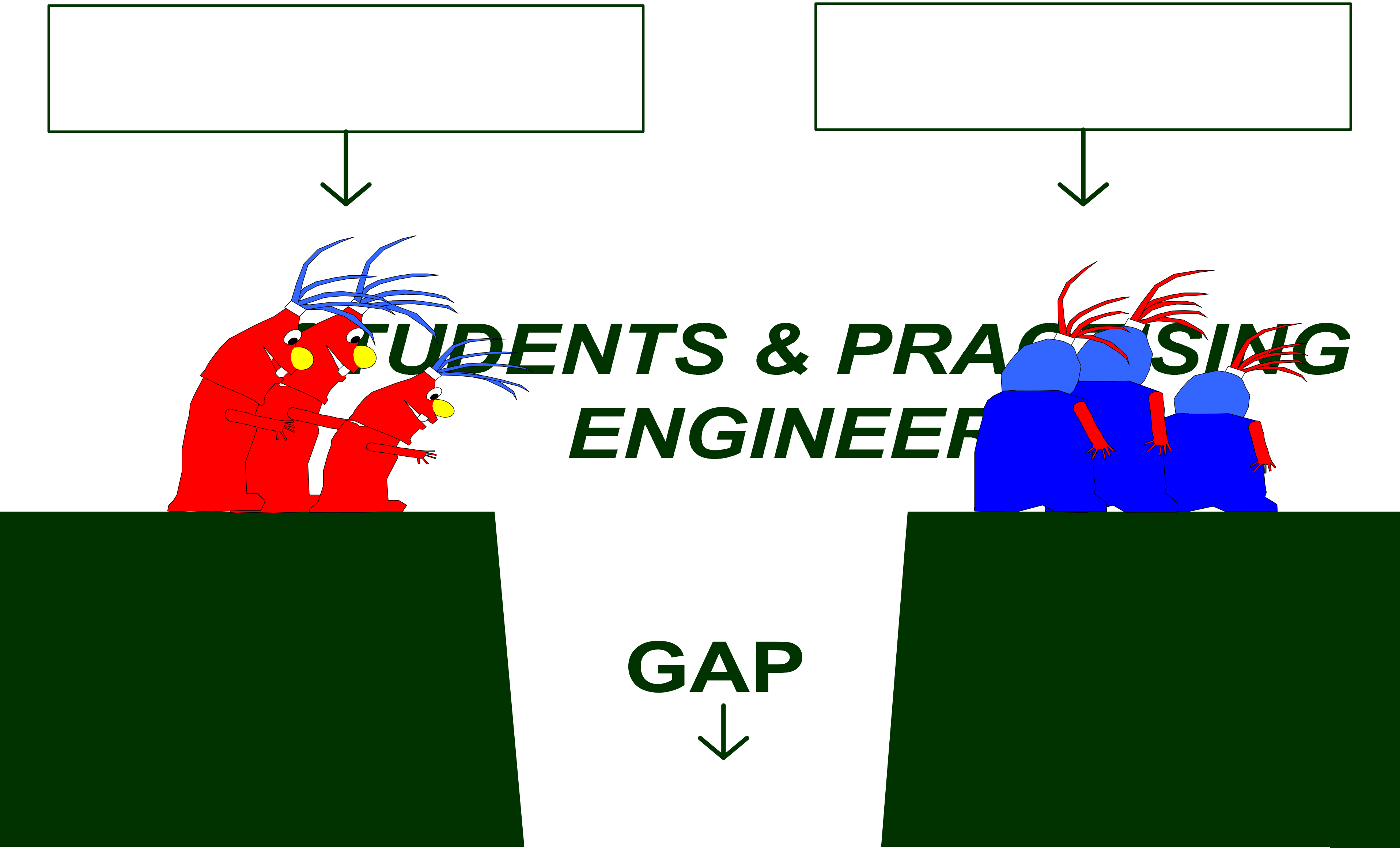
I. Structural reliability 8/10

Structural reliability theory: *arguments against*

- *The need to study probability calculus and statistics*
- *The need to collect statistical data on structures and actions (loads)*
- *The need to move outside the “safe and customary“ area ruled by design codes of practice*
- *Do you know the answer on the question “How safe is safe enough?” ?*

I. Structural reliability 9/10

The need to bridge a gap: *how to join quickly?*



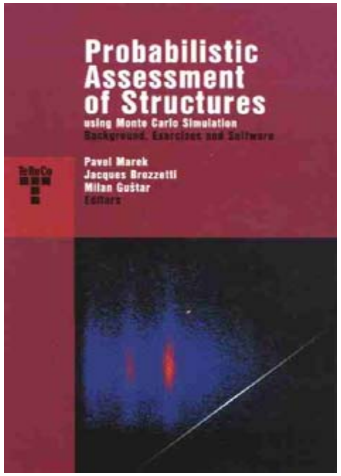
I. Structural reliability 10/10

The textbook

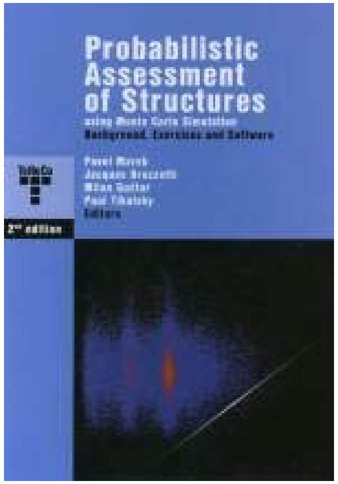
PROBABILISTIC ASSESSMENT OF STRUCTURES USING MONTE CARLO SIMULATION
Background, Exercises, Software
(*P. Marek, J. Brozzetti and M. Guštar, P.Tikalski, editors*)

Publisher: ITAM CAS CZ Academy of Sciences of the Czech Republic, Prague, 2001.

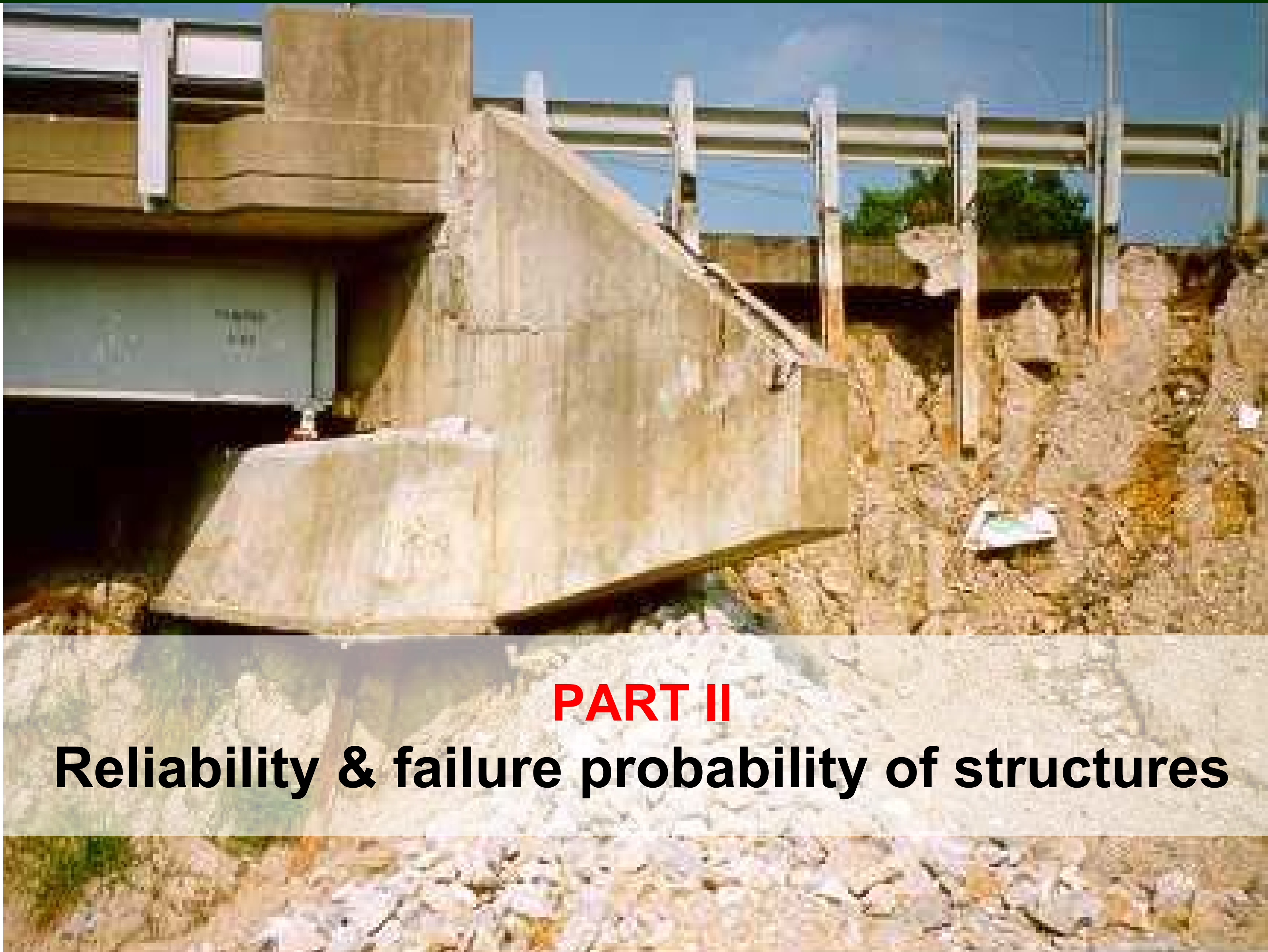
The textbook with the CD-ROM is the final product of a pilot project sponsored by the Leonardo da Vinci agency, European commission, Brussels, "TERECO - Teaching reliability concepts using simulation", 1999 - 2001.



For more information visit the website:
http://www.noise.cz/SBRA



Reliability & failure probability



PART II

Reliability & failure probability of structures

Basic definition

Reliability* = 1 – *failure probability

$$P_s = 1 - P_f$$

Structure can either fail or survive:

$$P_s + P_f = 1$$

- *Failure* * : an insufficient load-bearing capacity or inadequate serviceability of a structure or structural element.
- *Limit state*: a state beyond which the structure no longer satisfies the design performance requirements.

* ISO 2394: 1998 (E). General principles on reliability for structures. ISO, Geneve, 1998.

II. Reliability & failure probability 2/10

Reliability is usually not calculated!

SURVIVAL OF STRUCTURE (fulfillment of specified requirements)	FAILURE OF STRUCTURE (exceedance of irreversible or reversible limit state)
$P_s = P(\text{survival})$	$P_f = P(\text{failure})$

$$P(\text{survival}) + P(\text{failure}) = 1$$

$$P_s + P_f = 1$$

Structure 1	$P_{s1} = 0{,}9995$	$P_{f1} = 0{,}0005$
Structure 2	$P_{s2} = 0{,}999$	$P_{f2} = 0{,}001$
Difference	0,05%	200%

II. Reliability & failure probability 3/10

How safe is safe enough?

Tolerable failure probabilities given in ENV 1991-1*

Limit state	Tolerable failure probability (design working life)	Tolerable failure probability (one year)
Ultimate	723×10^{-7}	13×10^{-7}
Fatigue	$0{,}0668 \ldots 723 \times 10^{-7}$*	~---
Serviceability	0,0668	0,00135

* Depends on degree of inspectability, reparability, and damage tolerance.

* ENV 1991-1: 1993. Basis of design and actions on structures. CEN, Brussels, 1993.

II. Reliability & failure probability 4/10

The way to failure probability

Statistical data on structural parameters and loads

↓

Processing the statistical data

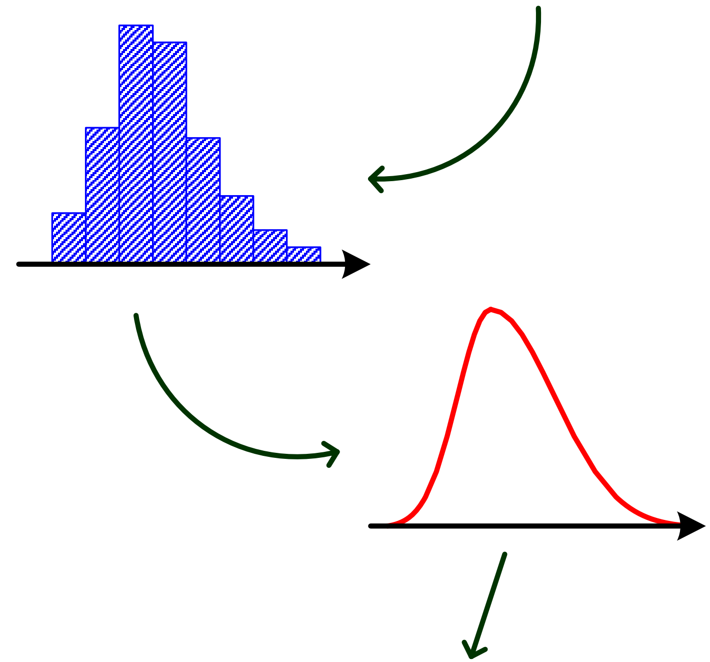
↓

Mechanical model of structure

↓

Estimation of failure probability
P

25,1; 33,2; 21,7; 30,3; ...



$$\frac{\text{no of failures}}{\text{no of trials}}$$

The role of data

- **Statistical data reflects the ubiquitous *uncertainty* in structural parameters and loads.**
- **Statistical data is used to fit *probability distributions* of the structural parameters and loads.**
- **Statistical data *determine* the value of the failure probability P_f in the end.**

- **Data on *material properties***
- **Data on *geometrical quantities***
- **Data on *direct actions* (*loads*) and *indirect actions***
- **Data on *model uncertainties***

Processing the data

x_1

x_2

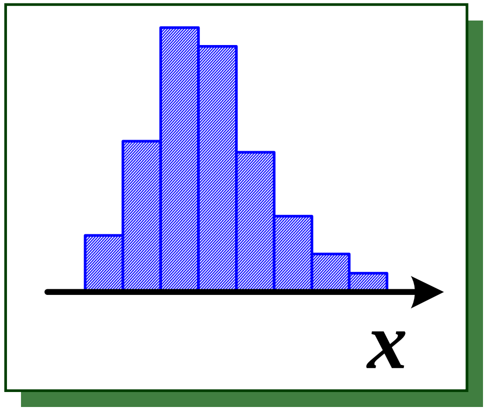
x_3

x_4

$\vdots$

x_{n-1}

x_n



$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

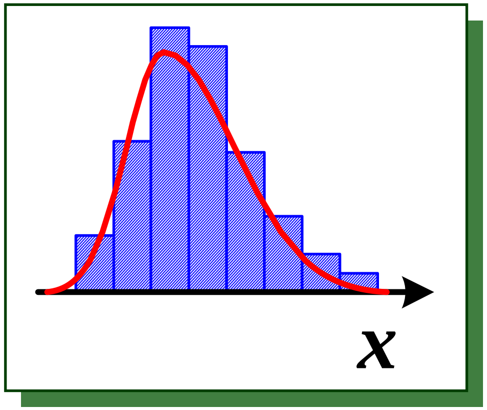
$$\delta = \frac{s}{\bar{x}}(100\%)$$

Estimation of distri-bution parameters:

$$\mu = \bar{x}$$

$$\sigma^2 = s^2$$

Fitting a probability distribution



Vector of basic variables

$\boldsymbol{X} = (X_1, X_2, \dots , X_n)$; $\boldsymbol{x} = (x_1, x_2, \dots , x_n)$

random variables **particular values**

Visualisation of X and x

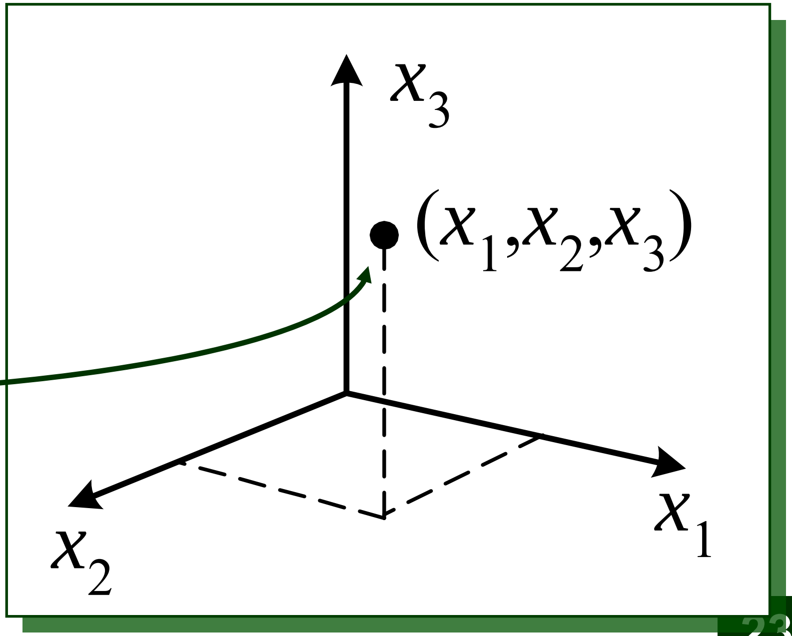
The case $n = 3$

$\boldsymbol{X} = (X_1, X_2, X_3)$

three-dimensional space

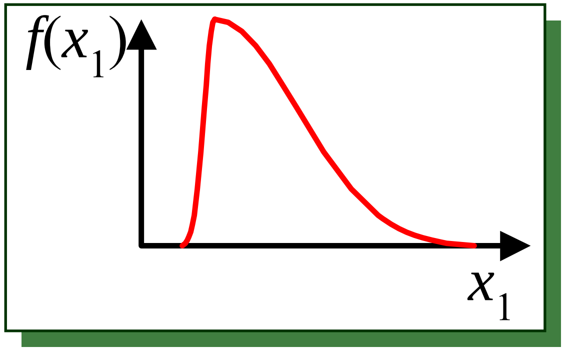
$\boldsymbol{x} = (x_1, x_2, x_3)$

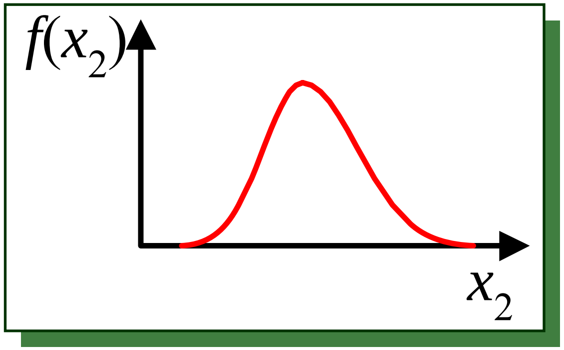
point in the space



II. Reliability & failure probability 8/10

Probability density function (PDF)





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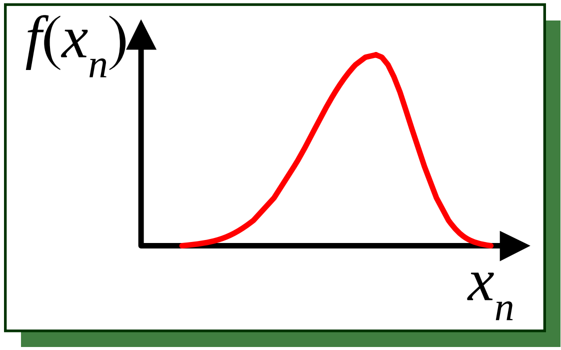


univariate PDF of X_1 | univariate PDF of X_2 | ... | univariate PDF of X_n

Joint PDF of basic variables

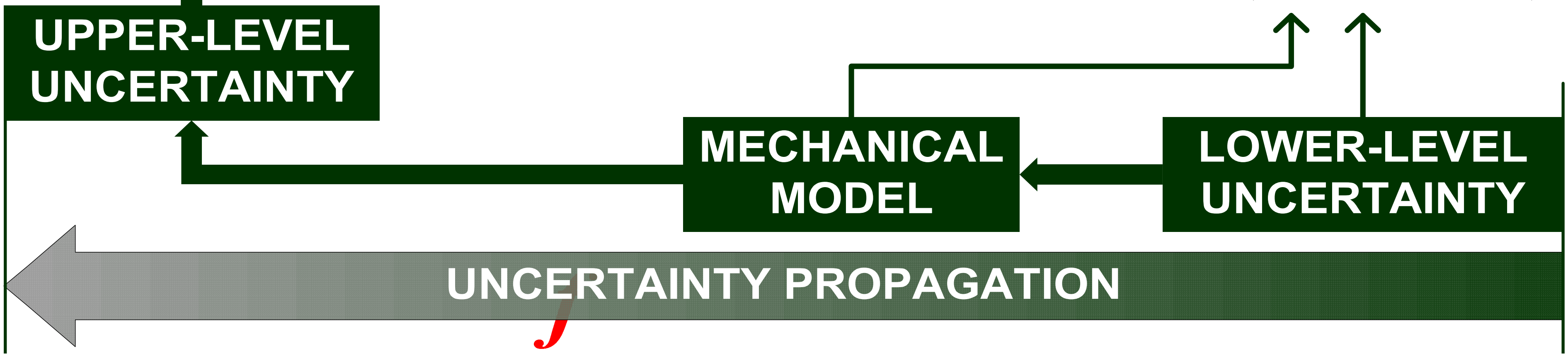
$$f(\mathbf{x}) = f(x_1) \cdot f(x_2) \cdot \ldots \cdot f(x_n)$$

in the case that basic variables X_i are *independent*

It is nothing more than uncertainty propagation!

$$P \; = P(\text{ Random Safety Margin } \quad 0) =$$

$$= P(\, g(X) \quad 0 \,)$$



- *Limit state function*: a function g of the basic variables, which characterizes a limit state when $g(x_1, x_2, \ldots, x_n) = 0$; $g > 0$ identifies with the desired state and $g < 0$ with the undesired state [state beyond the limit state].
- *Basic variable* X_i : a part of a specified set of variables, $X_1, X_2, \ldots, X_n$, representing physical quantities which characterize actions and environmental influences, material properties including soil properties, and geometrical quantities.

The problem is an integral, not the reliability itself!

$$P = P\left(g(X) \quad 0 \right) = \int_{D_f} \ldots \int f_X(x)\,\mathrm{d}x$$

Failure domain

Joint PDF

$$D_f = \{x \mid g(x) \leq 0\}$$

$$x_1, x_2, x_3, \ldots$$

The role of model

The role of data

Monte Carlo method

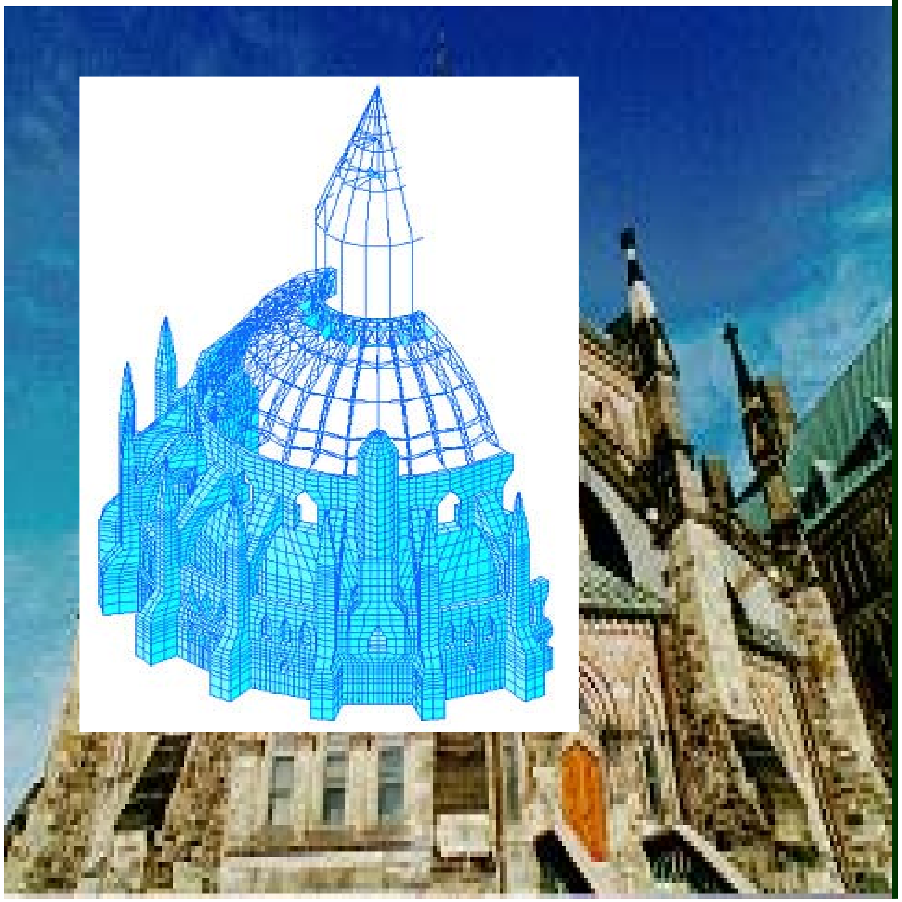
PART III

Monte Carlo method

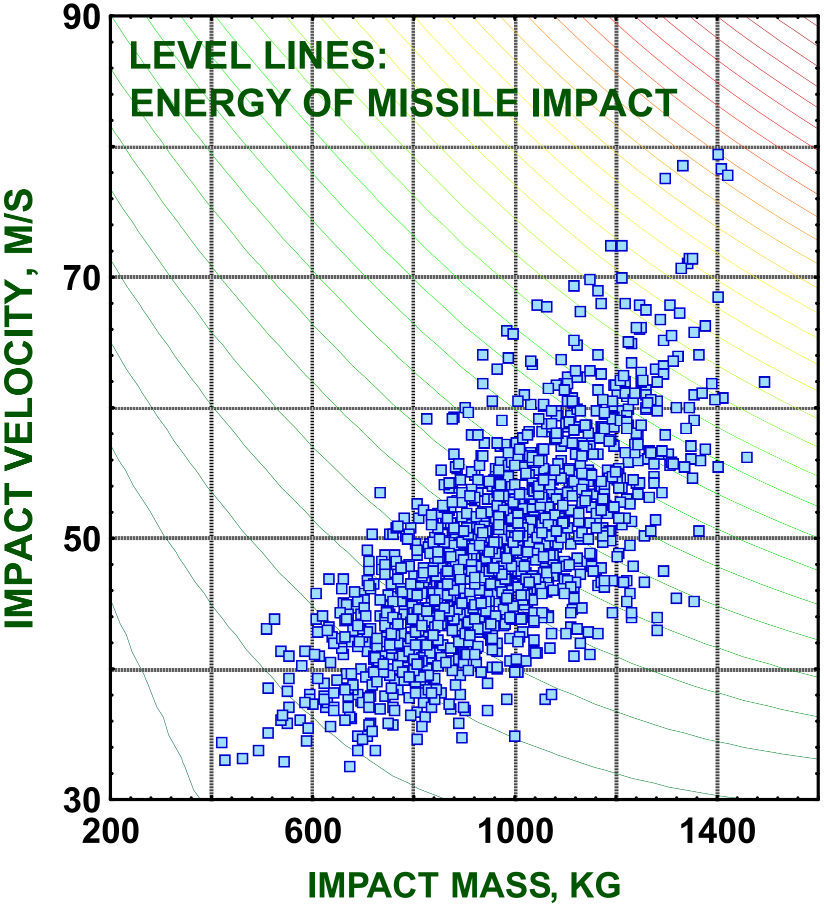
III. Monte Carlo method 1/10

The power of Monte Carlo method

MECHANICS
(ANALYSIS OF STRUCTURES)



STOCHASTICS
(PROBABILITY CALCULUS)



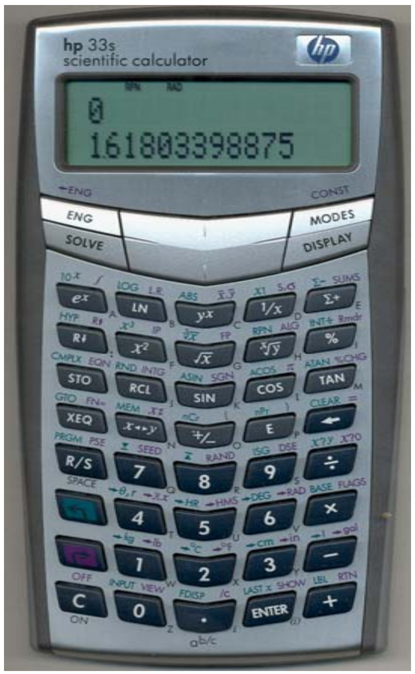
III. Monte Carlo method 2/10

Where can it be useful?

- **Structural Reliability**
- **Structural aspects of risk analysis**
- **Solving special problems, e.g., sensitivity analysis**

Only one small "detail" is necessary to run the business!

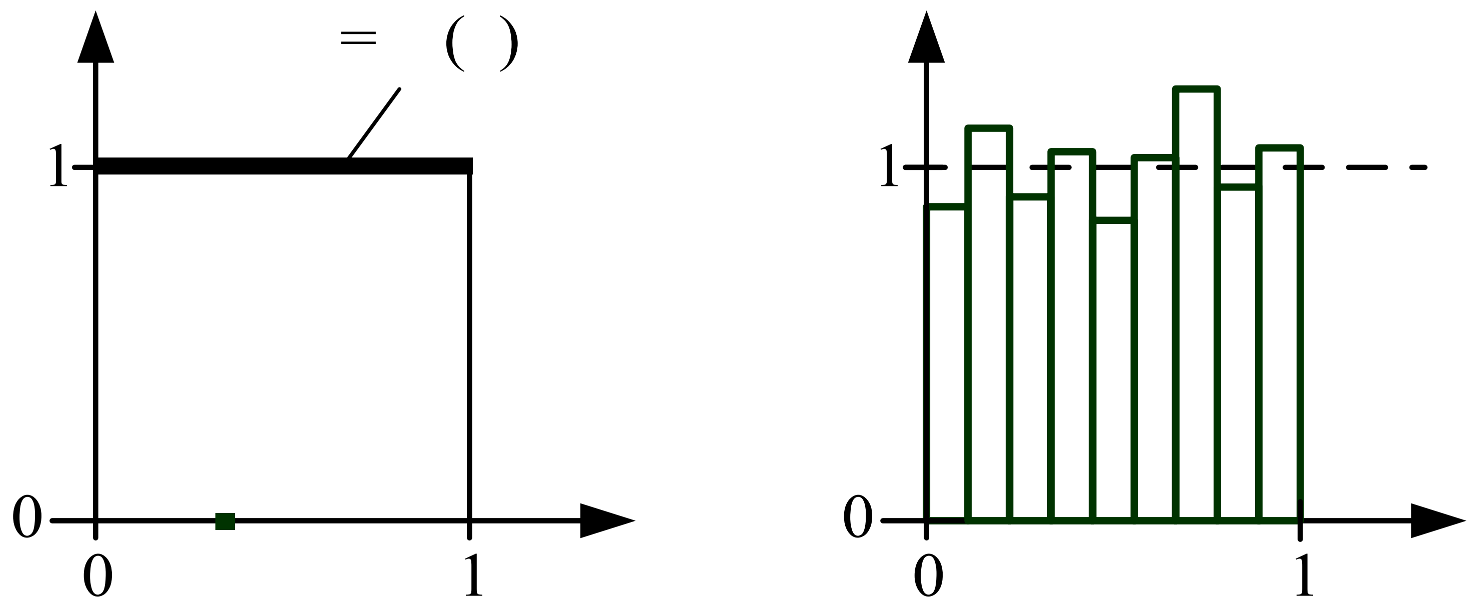
1. Generator of random numbers is in your *pocket calculator*



$$u_1 = 0.405255$$
$$u_2 = 0.764611$$
$$u_3 = 0.041139$$
$$u_4 = 0.013642$$
$$\vdots$$

Only one small “detail” is necessary to run the business!

2. Generating values from the uniform distribution U(0, 1)



III. Monte Carlo method 5/10

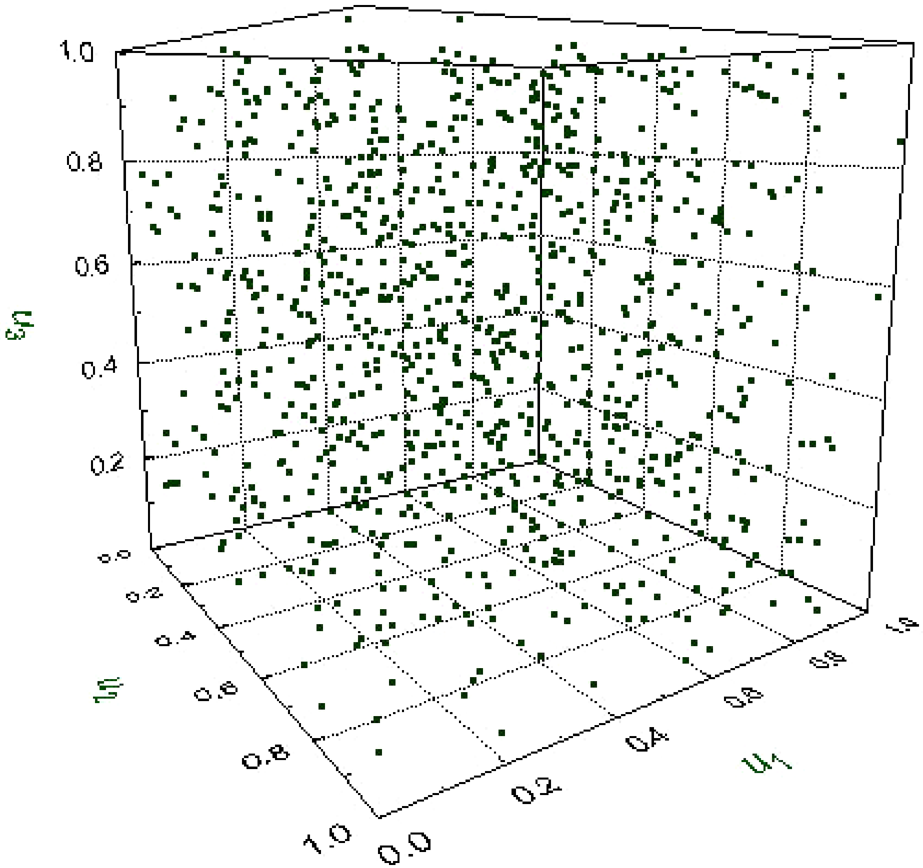
Only one small "detail" is necessary to run the business!

3. Generalization to the multidimensional case is beneficial

Evaluating multidimensional integrals & solving integral equations

Solving systems of differential equations

Solving systems of linear equations



See, e.g., Rubinstein, R. Y. (1981) Simulation and the Monte Carlo method, Willey.

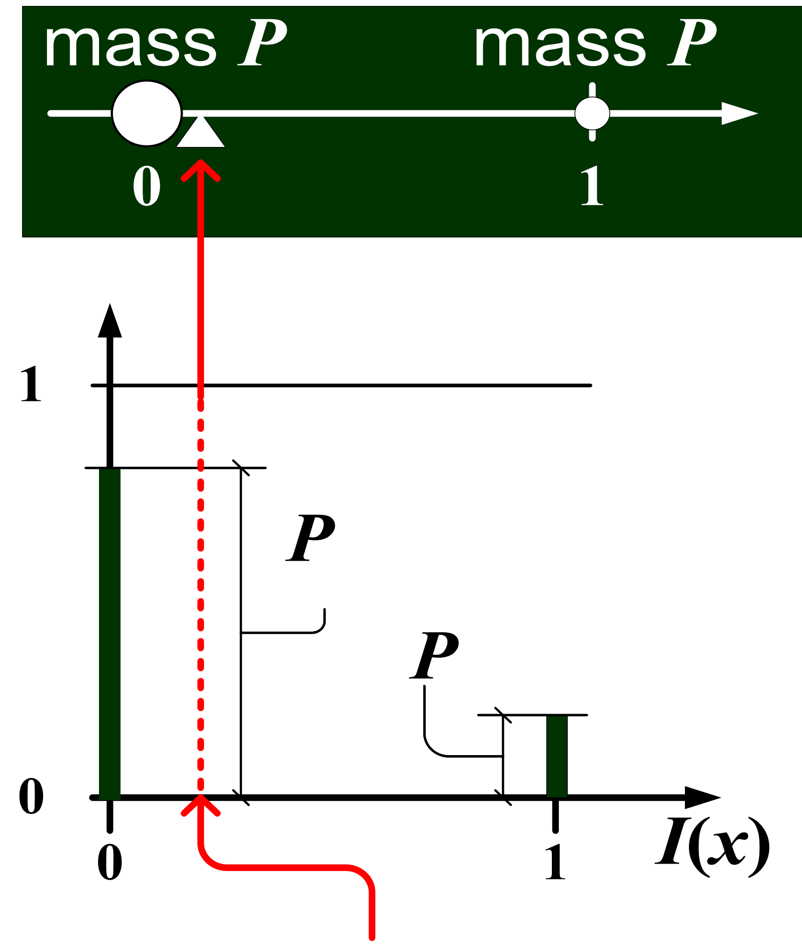
How to evaluate multiple integral?

$$P_f = P\big(g(X) \le 0 \big) = \int \underset{D_f}{\dots} \int f_X(x)\,\mathrm{d}x$$

- Exact analytical methods
- Classical methods of numerical integration
- Approximate analytical methods (FORM/SORM methods)
- **SIMULATION (MONTE CARLO) METHODS:**
  - **Direct Monte Carlo method**
  - **Variance reduction techniques**
  - **Methods utilising knowledge on mechanical model (response surface method, directional simulation)**

Failure probability is a mean of random variable

$$I(x) = \begin{cases} 1 \text{ if } g(x) \le 0 \text{ (failure)} \\ 0 \text{ if } g(x) > 0 \text{ (survival)} \end{cases}$$



Vector of random arguments X

Probability mass function of $I(X)$

$$P = P(I(X)=1) = \int \cdots \int_{\text{all } x} I(x) f_X(x) \, \mathrm{d}x = E(I(X))$$

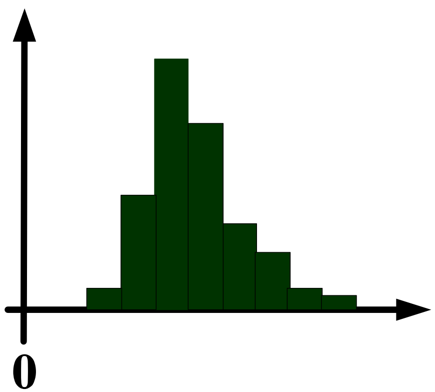
III. Monte Carlo method 8/10

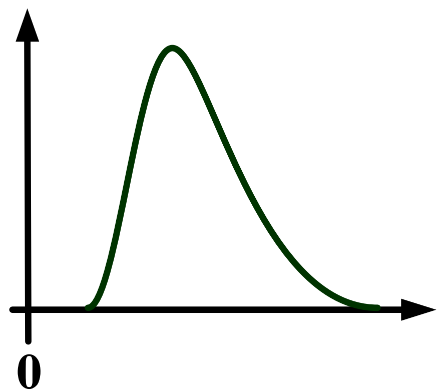
Estimate of failure probability

Intuitive definition → $P_{fe} = \dfrac{\text{Number of failures } N_f}{\text{Number of trials } N}$

Formal definition → $P_{fe} = \dfrac{1}{N}\sum_{j=1}^{N} I(x_j)$ → $E(I(X)) = P$

Collect original data sample → Fit PDF → Generate sample from PDF → Compute sample of 0s and 1s

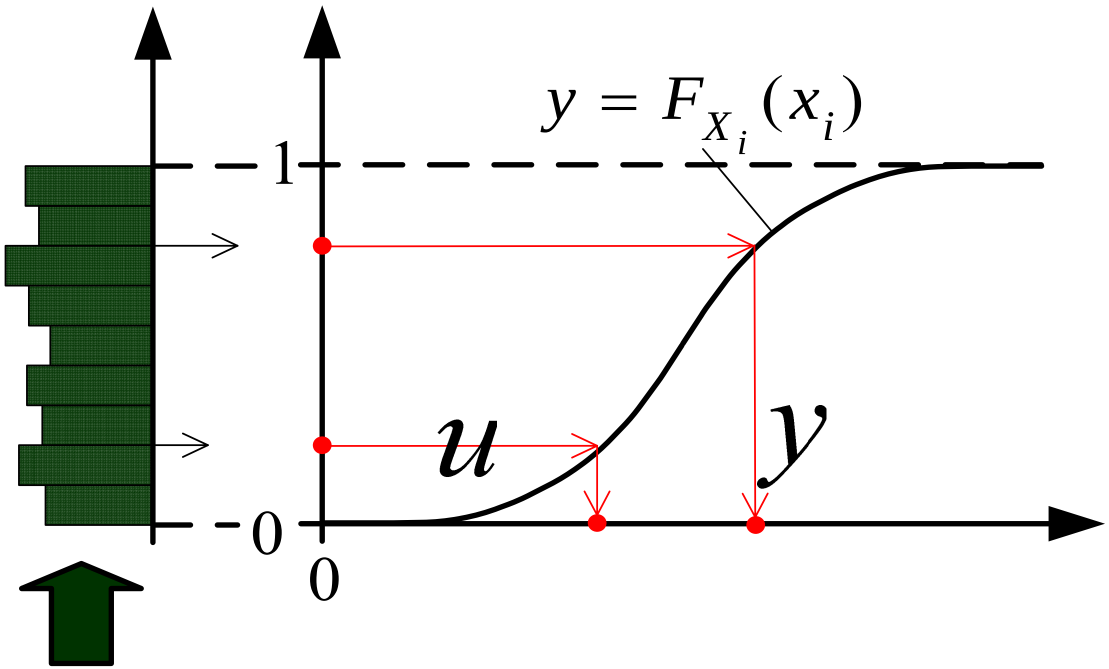


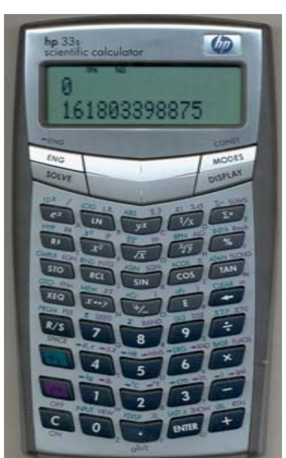


x ,
x ,
x

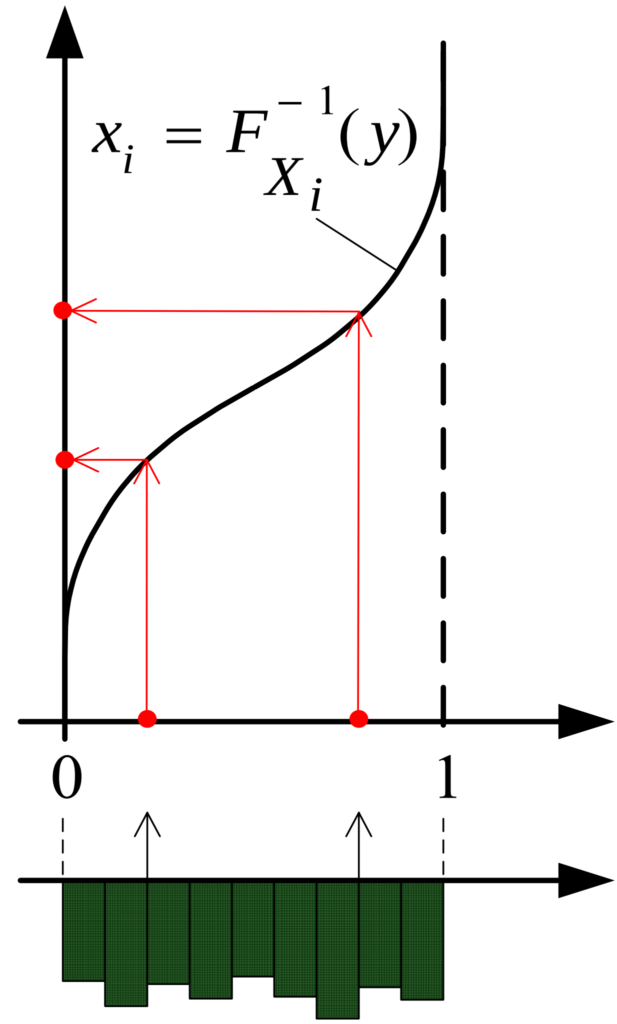
$I(x\ )$,
$I(x\ )$,
$I(x\ )$

Generating values of basic variables: inverse transform





$$x_{ij} = F^{-1}_{X_i}(u_j)$$



III. Monte Carlo method 10/10

Generating values of basic variables: in general

Generating *individual values* of basic variables

Extreme value distributions	Inverse transform method
Exponential, *Pareto*-, *Raleigh*-, *Cauchy*-distributions	Inverse transform method
Normal distribution	Composition method, other methods
Gamma distribution	Acceptance-rejection method
Beta distribution	Composition method, other methods
Lognormal distribution	Simple transformation from normal
Truncated distributions	Acceptance-rejection method

USUALLY HIDDEN IN COMPUTER CODES

Generating *vectors* of basic variables

Multi-normal distribution	Special method
Non-normal vectors with correlated components	Methods based on transformations from normal
General case of dependence	Multivariate transformation method, multivariate acceptance-rejection method

USUALLY HIDDEN IN COMPUTER CODES

For more visit, e.g., http://random.mat.sbg.ac.at/literature/

The end of theoretical part